# *The Fundamental Theorem of Algebra*

An *n*th degree polynomial function has exactly *n* zeros. *

$f(x) = 3x^4 - 2x^3 - 4x + 9$ has 4 zeros

$f(x) = 2x^6 + 5x^5 - 4x^4 + 9x^2 + x - 7$ has 6 zeros

$f(x) = -2x^2 - 4x + 9$ has 2 zeros

$f(x) = 3x^2 - 2x^3 - 4x^5 + 9x - 3$ has 5 zeros

* Each repeated solution must be counted as a separate solution.

$$f(x) = x^2 + 4x + 4$$
$$= (x+2)(x+2)$$

$$x = -2$$

Find all the zeros of $f(x) = x^5 - 4x^3 + x^2 - 4$

**1. List possible zeros:** $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1} = \pm 1, \pm 2, \pm 4$

**2. Test possible zeros (look at graph first if you want to.)**

$$\begin{array}{r|rrrrrr}
1 & 1 & 0 & -4 & 1 & 0 & -4 \\
 & \downarrow & 1 & 1 & -3 & -2 & -2 \\
\hline
 & 1 & 1 & -3 & -2 & -2 & -6
\end{array}$$

$$\begin{array}{r|rrrrrr}
-1 & 1 & 0 & -4 & 1 & 0 & -4 \\
 & \downarrow & -1 & 1 & 3 & -4 & 4 \\
\hline
 & 1 & -1 & -3 & 4 & -4 & \cancel{0}
\end{array}$$

$$\begin{array}{r|rrrrr}
-1 & 1 & -1 & -3 & 4 & -4 \\
 & \downarrow & -1 & 2 & 1 & -5 \\
\hline
 & 1 & -2 & -1 & 5 & -9
\end{array}$$

-1 is not a repeated solution.

$$\begin{array}{r|rrrrr}
2 & 1 & -1 & -3 & 4 & -4 \\
 & & 2 & 2 & -2 & 4 \\
\hline
 & 1 & 1 & -1 & 2 & 0
\end{array}$$

$$\begin{array}{r|rrrr}
2 & 1 & 1 & -1 & 2 \\
 & & 2 & 6 & 10 \\
\hline
 & 1 & 3 & 5 & 12
\end{array}$$

$$\begin{array}{r|rrrr}
-2 & 1 & 1 & -1 & 2 \\
 & & -2 & 2 & -2 \\
\hline
 & 1 & -1 & 1 & 0
\end{array}$$

$$x^2 - x + 1$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

Zeros

$-1, 2, -2, \dfrac{1 \pm i\sqrt{3}}{2}$

p. 374/ 19 - 28, 45, 46, 49, 50 due Wed, Jan 15 (5.6)

Take-home quiz (5.1 - 5.6) due Friday